APEC Seminar on Best Practices and Innovations in the teaching and learning of Science and Math:

Improving Math Education in Chile: Standards, e-Tutoring and Multiplayer Games

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Abstract:

The country is nowadays performing an important effort to radically improve the teaching of mathematics. For the first time standards with progress maps are being developed. New strands such as statistics and probability, and mathematical reasoning are being introduced. Also several new approaches such as vector geometry and new topics such as model building are being included. At the same time a couple of private and public projects are developing and implementing innovative didactic materials, software and on-line multiplayer games to teach algebra, statistics, geometry and math reasoning. Also new teaching services to students through internet based interactive tutorials and e-learning programs for teacher training are being offered.

In the first part of this presentation a broad summary of the present status of the development of the Math Standards will be shown. In the second part several initiatives of educational services, such as student e-tutoring and teacher training through e-learning, will be briefly described. In the third part several developments of internet based math teaching multiplayer games will be presented and some preliminary results of the use of such teaching innovations will be commented.

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Standards

For the first time standards are being developed in the country. Five strands are being proposed for Math: Numbers, Algebra, Geometry, Information and Randomness, and Mathematical Reasoning. Two of them include contents and performances that are almost completely new for the national education system.

One is Information and Randomness. The corresponding contents were previously partially present on the national curricula but only on the last three grades and only in a very superficial form. The other one is Mathematical Reasoning. This is even a bigger change. Previously, there were only some very general comments in the curriculum asking for some kind of preoccupation to promote general mathematical and logical reasoning. Now, on the new proposed standards, a great effort has been invested on trying to specify the type of contents, performances and progress expected on this strand.

Numbers

It includes four growth dimensions:

- Number understanding, its uses, and the structure of the different numerical systems from natural numbers to complex numbers.
- Order and Metric, spatial representations, and limits of sequences. Regarding the real numbers, this strand will emphasize the understanding of the idea that all real numbers, whether they are rational or not, can be approached as closely as desired by rationals.
- Counting, from basic counting to some elementary combinatorics.
- Operations, algorithms, both mental and written computations.

Algebra

Algebra is introduced now from grade 1. This starting at an early grade is completely new. Literals start on level 2, and matrix algebra on level 6. There are four growth dimensions:

- Equation solving: write and then solve equations.
- Algebraic structures: How operations provide structure to the elements.

- Patterns and functions: recognize and describe patterns and functions.
- Symbolic language, as an extension of natural language to describe more precisely general situations.

The following example illustrates and indicator about the use of matrices on the Patterns and Function growth dimension on level 6:

Indicator: Show properties and obtain results with matrices seen as linear functions.

Example: Show that if A is a $2x2$ *matrix then the function* $F(x,y) = A\begin{pmatrix} x \\ y \end{pmatrix}$ *satisfies* $F(\lambda(x,y)) = \lambda F(x,y)$ *and* $F((x,y)+(a,b)) = F(x,y)+F(a,b)$. *And, conversely, show that if a function from the plane into itself satisfies the two previous conditions then necessarily the function is a multiplication by a matrix.*

Geometry

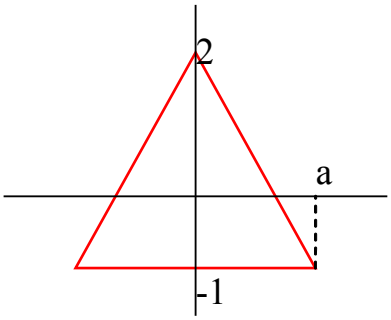
There are three growth dimensions:

- Location: position and spatial movements
- Form: classification and recognition in real world situations
- Size: one, two and three dimension content measures and angle.

The proposed Standard place an emphasis on using vector language in Geometry. For example, here is an indicator and an associated example of the Location growth dimension on level 5.

Indicator: Describe symmetries, presenting translations, rotations and reflections in vector form.

Example: in the equilateral triangle of the figure, if $a = \sqrt{3}$



determine its axes of symmetry, with vector calculation of the reflections about them, and check that the triangle does not change when these reflections are applied . Determine also the rotations that do not alter the triangle, and perform vector calculations to check your answer. Show that the composition of these rigid motions leave the triangle fixed, as do their inverses

Information and Randomness

In the new proposed Standard the strand Information (Statistics) and Randomness (Probability) is introduced from grade 1. This is completely new since on the previous curricula these domains were only taught from grade 9 on. Also now the intention is to provide an integrated approach of probability and statistics. Additionally, there is also the objective to teach and practice real world applications.

There are two growth dimensions:

- Organize, interpret and infer information
- Justify and formalize inferences

The following example is part of an example illustrating an indicator for the Justify and Formalize Inferences growth dimension on level 7:

Indicator: Comparing cumulative histograms, compute the power of different variables to discriminate between two factors, using as a measure of discrimination of a variable the maximum vertical distance between the cumulative histogram of one factor and the cumulative histogram of the other factor, and as a cut point the value of the variable where the maximum is reached.

Example: compute the power of temperature to discriminate between healthy and sick patients with a

cold. Use the data of a sample of healthy and sick patients as graphed on the two cumulative histograms shown, one is the cumulative distribution of the temperature of healthy patients and the other one is the cumulative distribution of the temperature of patients with cold. Use as discrimination measure the maximum vertical distance between the two cumulative histograms.

Cumulative distributions of temperatures of healthy (bold line) and sick patients

Mathematical Reasoning

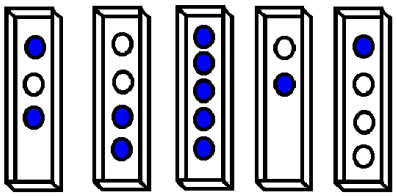
Four growth dimensions are considered:

- language,
- modeling,
- strategies, and
- argumentation.

The Language growth dimension includes the geometric, algebraic, probabilistic languages, and the languages of logic and sets. Here is an example of the growth dimension language at level 2:

Indicator: Reads and interpret sentences with instructions with two or more logic connectives "and, "or", or mixes of both, two or more quantifiers and set unions and intersection symbols.

Example: Look at the picture and say if true or false: "there is one box in which all marbles are of the same color".



The Modeling growth dimension includes pattern description, problem solving, building math models and computer simulations. This is an example on level 7:

> *Indicator: Explore model of dynamic phenomena in different scientific domains: physics (movement of particles), biology (neurons, populations), psychology (habituation, hunger), social sciences (migrations) and economy, building simulators, analyzing the results and determining what parts of the phenomena were well captured.*
>
> *Example: In the habituation process the response to a stimulus depends on how many recent stimulations the agent have received. Just after a trend of stimulus the response becomes weaker. But, after some time with no stimulus then the original response strength is recuperated. If u(n) is the stimulus at time n (0 if no stimulus and 1 if stimulus is applied), and if y(n) is the agent response at time n, then show that the following model*
>
> $y(n)=max(0 , u(n)-0,2x(n))$
> *where*
> $x(n)= 0.5x(n-1)+u(n-1)$
> $x(0)=0$
>
> *corresponds to an habituation process by computing and verifying that the response to three consecutive stimulations becomes weaker each time, but that the strength of the response is almost recuperated after some time with no stimulus. How would you change the 0.5 parameter so that the impact of stimulation last longer?*

The Strategies growth dimension includes several metaphors, like the one of a machine for a function and the rotation and scaling for multiplication. It also includes metacognitive strategies for making conjectures, analogies, approximations, and strategies for verification and sensitivity of results.

The Argumentation growth dimension includes following procedures, analysis of algorithms, and making chains of deductions and proofs.

Standards for teacher education in Math

These standards are also in the process of development, by a FONDEF[1] project team led by the University of Chile. This is the first effort in the country to develop such type of standards. They are part of an initiative that also includes science standards. The strands and subs strands are:

Algebra and Discrete Structures
- Logic, Sets and Discrete Structures
- Algebraic Structures
- Linear Algebra

Analysis and Geometry
- Analysis
- Geometry

Probability
- Probability
- Statistics

There will be also a Scientific Method strand that will be developed together with Science. The progress map for each strand and its corresponding growth dimension is organized on four levels.

Tutoring Services and Teacher Training

Tutoring directly to students

A new kind of initiative being developed is the development of tutoring services provided by universities and reaching directly to high school students. The first such program is being implemented by the University of Chile, through the FONDEF[1] project "Interactive Tutoring in Mathematics". At the present time three interactive tutoring services for High School Mathematics are in preparation. They are for the areas of Algebra, Geometry and Probability. They include tutoring materials provided through the internet and the multiplayer didactic games.

[1] FONDEF is a Fund for the Promotion of Scientific and Technological Development, which was founded in 1991 as a direct government initiative to improve the level of R&D. CONICYT is the administrator of this fund.

Tutoring using the growing home access to broadband internet

Internet services have experimented an enormous grow the last couple of years. Approximately 11 % of homes have broadband internet connection, and at the present grow rate in three years 50% of homes will have broadband internet connection. This trend has sparked the offering of didactic services for students at home, both from public and private providers. Still the offering in math is limited but an interesting trend is underway, and the tutoring.

Outreach programs for talented high school students

Our students come from many schools disseminated throughout our very long territory, longer than going from San Diego to Boston. The quality of the education they bring with them depends mainly on their school of origin, which varies significantly. There are several initiatives being implemented to capture and teach talented kids. Some of the ones organized by the Universidad of Chile are:

A Summer school for talented students
Physics via internet: a distance course for high school students
New science program for high school science.

Teacher Training

Several initiatives are being developed by the government, the universities, the local municipalities and private providers. Some of them are:

- E-learning: Distance Learning Program for High School Mathematics teachers. For example, Vector Geometry and Probability e-learning courses have been already taught by the University of Chile several times. There are several other initiatives of offering e-learning teacher training on math for high school. For example, recently University of Santiago trained 800 teachers on functions.
- Mathematics Specialization Internships managed by the University of Chile. Teachers representing all regions of Chile, and selected by national contest, participate in a one month long, 8 hours a day internship program of specialization in topics of Mathematics.
- Creation of a sustainable national network of 40 Teacher Centers, which provides continuous professional development for teachers, resource center with technical support, physical space for the development of current ministerial programs, and place of interaction and collaboration between teachers and Universities. The network will be articulated through a Central Contents Unit,

with the help of the Chilean Academy of Sciences and the University of Chile. In this unit the quality of contents to be introduced into the network is to be certified.

Internet based Multiplayer Games

"Games as teachers" has been for long time an attractive promise. Now with internet there is also the possibility of multiplayer games through the net. In this section some preliminary innovative initiatives to build and use multiplayer internet based game to teach math will be shown.

Public projects for middle and high school math teaching.

The government has funded several projects to develop educational software. University of Chile has obtained a couple of projects funded by FONDEF. The most recent one is developing several multiplayer games to teach high school mathematics. Some examples of such games are:

Logical reasoning game:
This is a betting game where every player has a bag with boxes that contains ball of different colors, and has to try to guess what pattern of balls have the other players on their bags. The game makes the student formulate their conjectures using logic connectives and quantifiers. They have to choose from a menu a combination of logic connectives, quantifiers, containers (bag and boxes) and balls to generate meaningful propositions.

Vector Algebra game:
This is a sailing race where the students have to input vectors in order to control their vessels. The game makes the students compute vectors and rotation matrices in order to generate a good vessel command. Careful computation of the rotation matrix is critical since the game has been designed so that without control the vessel behavior is erratic and with careful computation chances to win are higher.

Probability game:
This is a betting game where each player has to select a card from a maze so as to improve his chances to dominate over the other players selected cards. Each card has a probability distribution known by the players, and based on that information careful probability computations are critical to estimate which card is better to select in order to have a high probability to win.

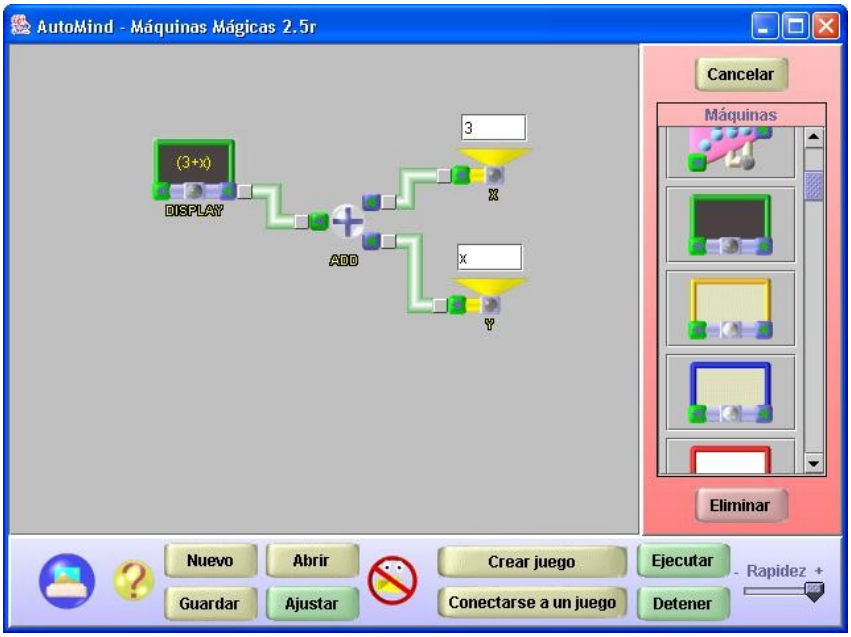
Private projects for middle and high school math teaching.

The government has partially funded several projects of educational software to teach mathematics that are developed by private companies. Here are shown some examples developed by AutoMind. Two of the games

were partially funded by FONTEC[2]. All of the games are multiplayer games where each player has to be connected to internet or to an intranet.

Function and function composition games

This software, called MAGIC MACHINES, uses the metaphor of a machine to visually define and build functions, composition of functions, dynamical systems, and perform simulations. The programming is made connecting different basic machines. For example, the following image shows the function f(x,y) = x+y, where the inputs are on the right and the outputs on the left, same as in the standard math notation for functions.



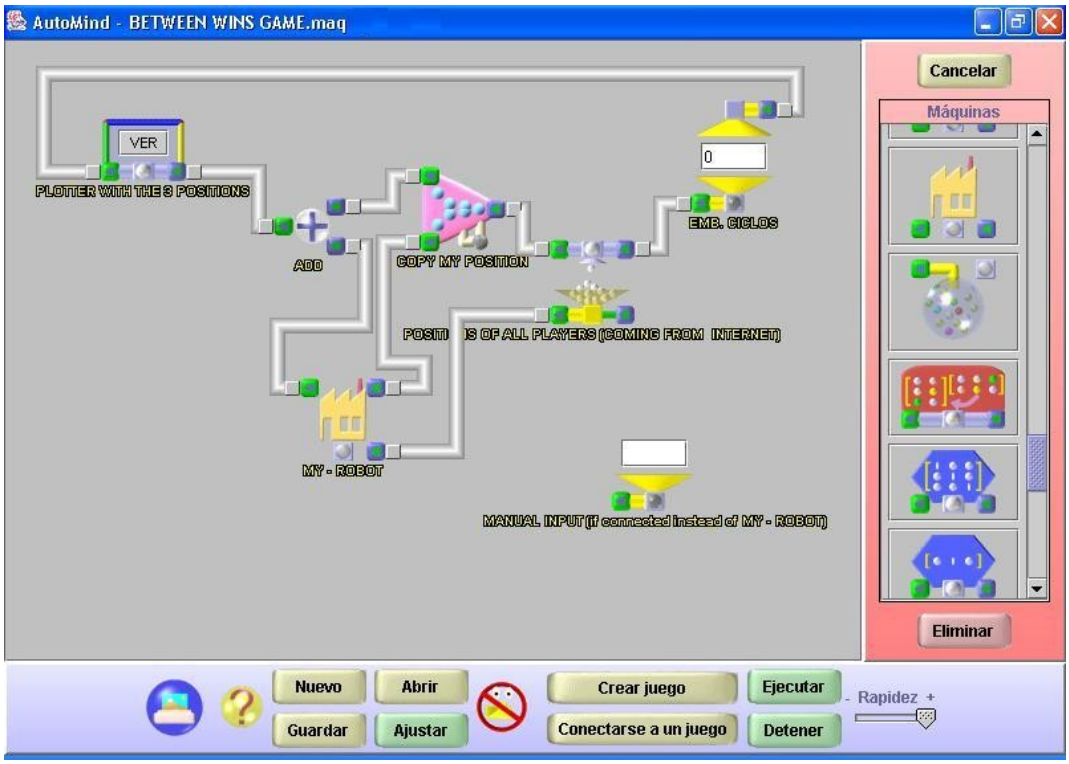
Several multiplayer games can be loaded on the MAGIC MACHINES environment. Three of them are shown next.

BETWEEN WINS GAME.

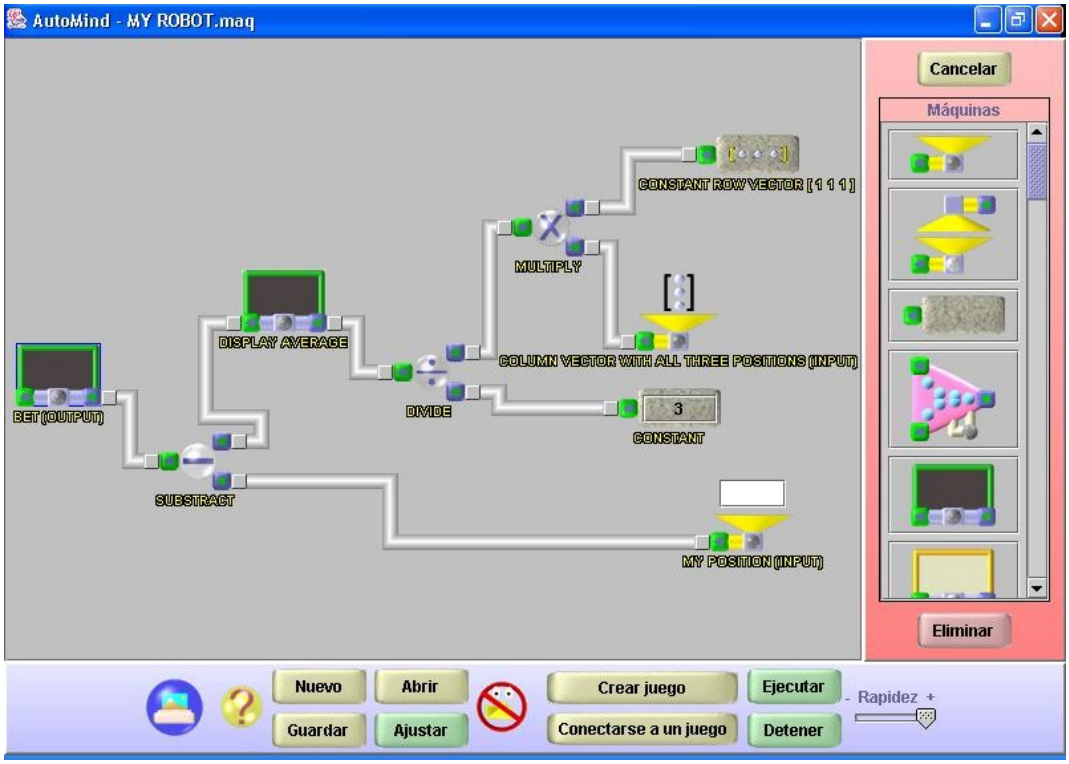
This is a game between three players where each one has to move a dot in a ruler. Each turn, the player whose point lies between the other two wins points, but if two positions are equals then both players lose points. Each player can choose to play entering numbers each turn or can bet with his own automatic betting machine (his robot).

[2] FONTEC is a government fund created to promote and cofinance projects of technological innovation. Corfo is the administrator of this fund.

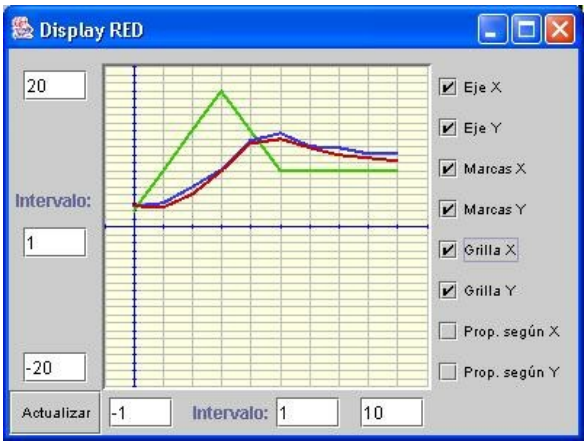
The following picture shows the BETWEEN WINS GAME loaded by the server where a MY – ROBOT automatic machine has been already placed by the player (if he wants to play manually then instead of placing a MY – ROBOT machine and connecting it to the lower entrance of the ADD machine, he would have to connect the MANUAL INPUT machine to the ADD machine).



The next screen shot shows the circuit of the MY – ROBOT machine built and loaded by the player. In this case his robot computes the average of the position of the three players and generates the difference of this number with his position.



The following graphic shows the positions through time displayed by the PLOTTER as a result of 10 turns where one player loaded the previous robot, another loaded a somewhat similar robot and a third player entered his bets manually.



Several other multiplayer games with the possibility of betting both manually or with automatic (robot) machines can be played in this environment. Next, two possible ones are described briefly.

BIGGEST ANGLE WINS GAME

This game is played by three players, each moving a point on the plane, and therefore in each turn a triangle is formed with the three

points. Each turn each player bet a vector to translate his point to another location. Each turn the player whose point is such that the angle on his vertex is the biggest of all three wins points, but if two points are equals then both players lose points.

PREY – DEPREDATOR RECTANGLES GAME

This game is played by three or more players, and each one has to move (translate and rotate) a 1x2 rectangle on the plane. Each player has associated one and only one prey and one and only one depredator. Each turn each player wins as many points as the area of the intersection of his rectangle with his prey rectangle, and lose as many points as the area of the intersection of his depredator rectangle with his rectangle.

The main goal of these didactic multiplayer games is to incentivate the use of algebra, functions and composition of functions, and use these concepts in order to capture the idea of strategy for decision taking and to specify a strategy in a precise and general form so that it can be implemented and put to work as in real world automatic machines.

Vector algebra and biological population modeling game

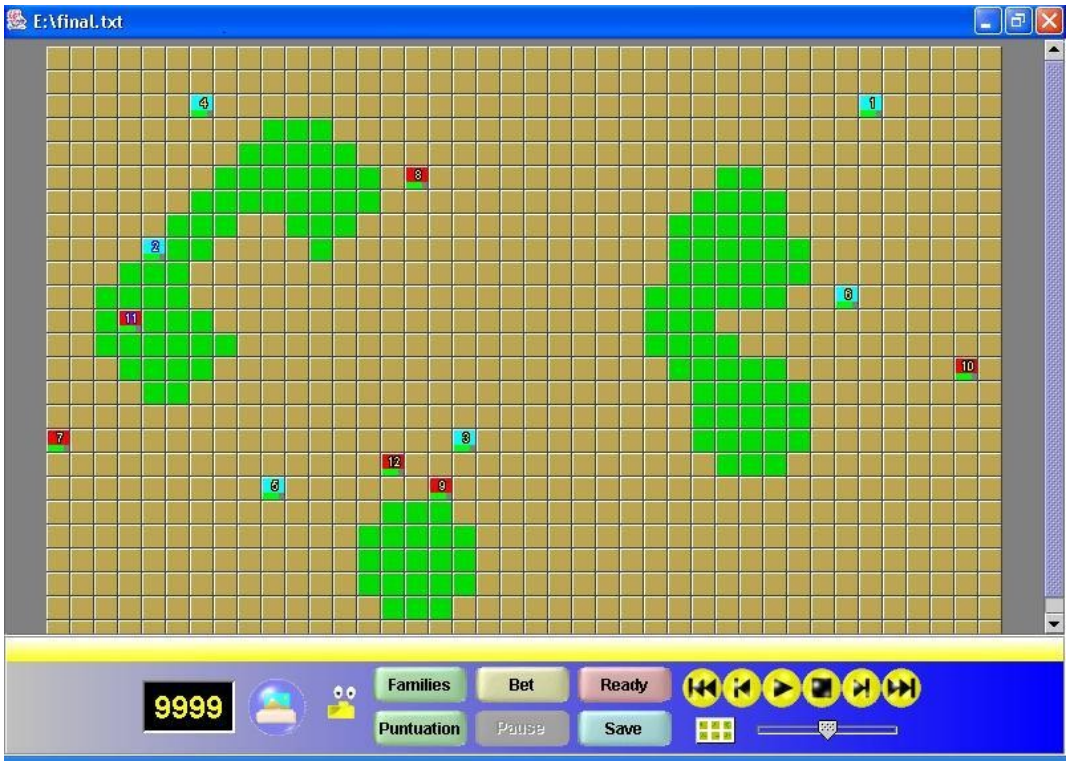
This is a multiplayer game, called MAGIC VECTORS, whose objectives are to learn and practice with the following concepts:

- Vectors and vector additions
- Inertia and particle motion physics (in discrete time)
- Prey – depredator dynamic and biological fitness modeling
- Social behavior modeling (kin selection, cooperation and reputation)

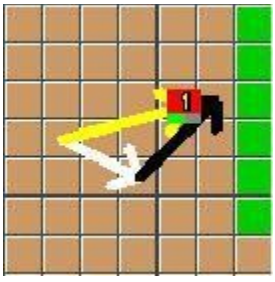
Some players are lions and others are zebras. There are two competitions going on: one between lions and one between zebras. After a fixed number of turns, the lion with more calories wins and the zebra with more calories wins.

Zebras gain calories eating grass. That means that at each turn that a zebra is at rest on a green square gain one calorie. A lion, however, gain calories eating zebras, and gain as much calories as the eaten zebra has. To eat a zebra the lion has to be no further than one square from the zebra.

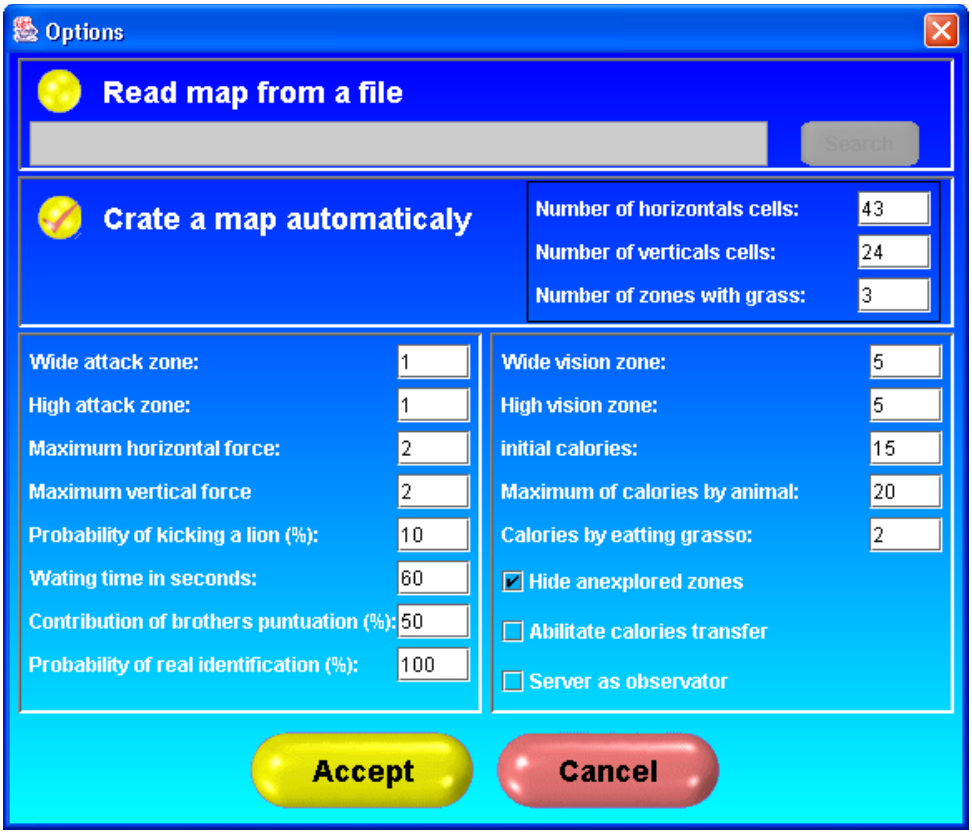
The following screen shot shows the board, where the green squares are grass, the brown squares are soil, the several numbered light blue squares are zebras and the numbered red squares are lions.



Each player can move his animal on the board betting for a force (a two dimensional vector limited to a certain range), and can also decide to give away some calories or punish some of the other animal of his species that are close enough. The motion of the animals is the addition of the force (black vector in the following portion of a screen shot) with the inertia (white vector), since it is assumed no friction.



The game has several options, as shown in the following picture of the option selection window:



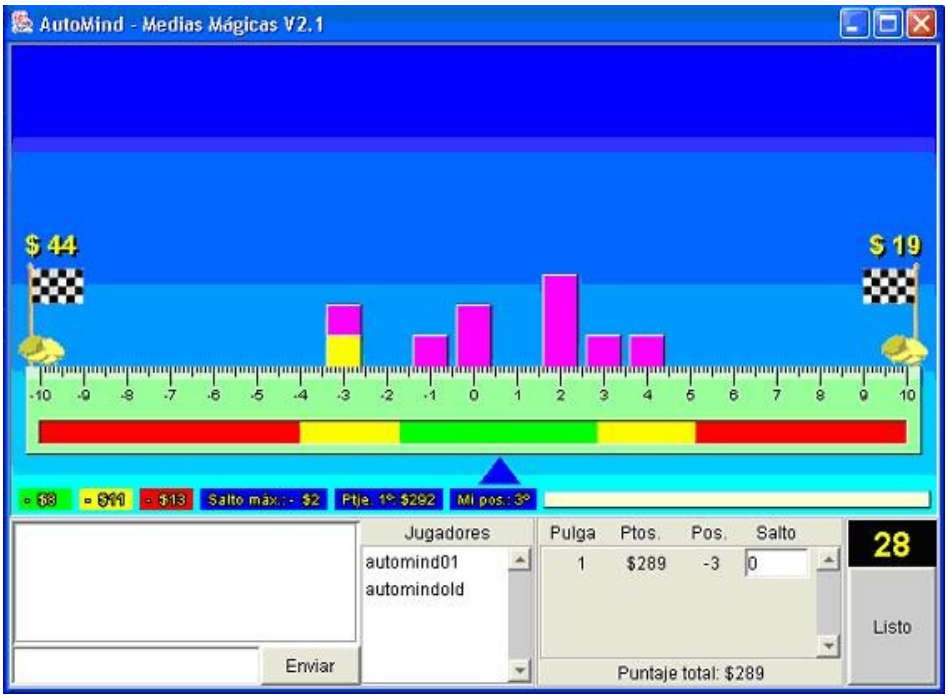
Statistical game: mean, standard deviations, histograms

The main objectives of this multiplayer game, called MAGIC MEANS, are to learn and practice:

- Reading and interpretation of histograms
- Visual estimation of the mean (just from looking to an histogram)
- Visual estimation of the standard deviation
- Estimate expected values
- Take decisions under risk
- Select strategies by predicting future means and standard deviations

In this game each player starts with a certain amount of virtual money and has to try to obtain more money by moving his flea over the ruler. If he reaches one extreme before anybody else then he wins the amount indicated on the corresponding flag and the round finishes and a new one starts. However, each jump cost money, and it is limited to two a fix maximum distance. Furthermore, the player also has to pay to park his flea and the price is different in different zones: the green zone (typically less expensive) is two standard deviation wide and centered on the mean, the yellow zone is between

one to two standard deviations, and the red zone (typically the most expensive one) is the one beyond two standard deviations. For students with more experiences, the mean and the zones are not shown. The game last 10 rounds, and in each round the prices changes randomly. The player with more money at the end of the 10 rounds wins.



Some preliminary results on student use of the multiplayer games

The games are still on the testing phase, both with students at home and in the school environment with students guided by teachers. We are gathering data on its use and impact in situations under our close monitoring and in independent use on class by teachers who report to us filling questionnaires.

MAGIC MACHINES has been used by elementary, middle and high school students, mainly building circuits that represent functions given by the teacher. The testing of multiplayer game capability is just starting. Teachers have received it with great enthusiasm. They have appreciated the list of activities and exercised that are provided with the game to check the student understanding. These are paper and pencil activities and quizzes that are designed for the different phases of the game use, covering from elementary use to more sophisticate use, and for different age groups.

MAGIC VECTOR has also been used by all class of student levels, from elementary school to college. Teachers also have appreciated

the list of activities and quizzes provided for the different levels of players and different levels of experience with the game and with its associated mathematical and modeling concepts.

MAGIC MEANS has only recently been used by some teachers. We are in the process of gathering data of its usefulness to student learning in real class environments, but laboratory measurements with students suggests that it should be so.

Preliminary comments by teachers and students confirm the impression that this is an interesting innovative strategy to teach math. It is fun, it helps to understand key concepts, and to see them applied in real situations. It helps to experience the concepts, practice them in reiterative but entertaining form. In other words, it helps to become familiarized with the concepts, to develop an appropriate mathematical intuition and to reach a more deep understanding of the mathematical ideas behind each game.

Besides the use of theses games by some Chilean schools, they are now included in a teacher training program offered by the Organization of American States (OAS) that will train teachers throughout all Latin America.